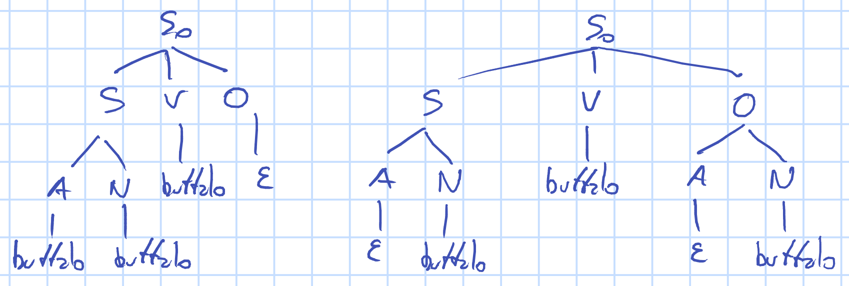
L8.

Grammars

The same string may be obtained in different ways. Then we say that a grammar is ambiguous.

e.g. $S_0 \to SVO$
$S \to AN$
$O \to AN \mid \varepsilon$
$V \to$ buffalo
$A \to$ buffalo $\mid \varepsilon$
$N \to$ buffalo



Some languages are inherently ambiguous.

e.g. $\{a^i b^j c^k \mid i=j \text{ or } j=k\}$.

Not all languages are context-free.

lem (Pumping): If L is context-free then $\exists p \in \mathbb{N}$ st if $s \in L$ and $|s| \geq p$, then $s = uvxyz$ st:

- $\to uv^ixy^iz \in L \quad \forall i \geq 0$
- $\to vy \neq \varepsilon$
- $\to |vxy| \leq p$

proof: See Sipser.

Using pumping lemma to show a language is not CFL:

$L_1 = \{ s \in \{a,b,c\}^* \mid \#a = \#b = \#c \}$

Proof: Assume L_1 CFL. Let p given by lemma.

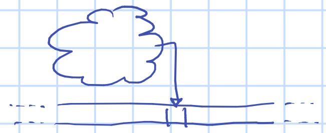
Choose $s = a^p b^p c^p \in L_1$.

$|vxy| \leq p \Rightarrow$ either no as or no cs.

By lemma, $uvvxyyz \in L$, but contains fewer as (in the first case) or fewer cs (in the second).

Contradiction!! Hence L_1 not CFL.

Turing Machines



TM = DFA + infinite tape.

- → input written on tape; blancks elsewhore.
- → initially pointing to first input character.
- → each step reads from tape, writes, and moves L/R.
- → two special "accept" & "reject" states.

e.g. A TM deciding $L = \{a^n b^n\}$.

Informal: move between as & bs, crossing them out alternately. When no longer possible, check all characters were crossed out.