L6. 2022-09-21

lem (Pumping) If L regular then $\exists p \in \mathbb{N}$ st if $s \in L$ and $|s| \geq p$, then $s = xyz$ st:

- $\to xy^i z \in L \quad \forall i \geq 0$
- $\to y \neq \varepsilon$
- $\to |xy| \leq p$

proof idea: an automaton is finite, so for large enough strings it will have to repeat states. Let y be the fragment of s we see during such a loop.

proof. Fix an automaton M recognizing L.

Let $p = |Q|$

Let $s \in L$ and $|s| \geq p$.

$\mathrm{comp}(M, p)$ is a sequence of $|s|+1 \geq |Q|+1$ states. By pigeon hole, there are two states $q_j, q_{j'}$ st $j < j'$ but $q_j = q_{j'}$ in the computation.

Pick $x = s_1 \dots s_j$, $y = s_{j+1} \dots s_{j'}$, $z = s_{j'+1} \dots s_n$.

Obs final state of $\mathrm{comp}(x) = q_j$

" " $(xy) = q_{j'} = q_j$

" " $(xyy) = q_j$

$(xy^i) = q_j$

Hence final state of $\mathrm{comp}(xy^i z)$ also the same $\forall i$.

Since xyz is accepted, so is $xy^i z$ $\forall i$.

For the last point, note we can assume wlog that $j, j' \leq p$.

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Pushdown Automata

If we add memory to an automaton, it can recognize more languages. If we allow any kind of memory we'll get Turing Machines. We'll discuss a more restricted model first: memory is a stack.

def PDA: NFA, with access to an infinite stack over alphabet Γ. Initial stack is empty.

Formally: $(Q, \Sigma, \Gamma, \delta, q_0, F)$

Q, Σ, q_0, F as usual. Γ finite set.

$\delta: Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \to Q \times (\Gamma \cup \{\varepsilon\})$.

not necessarily a function (i.e. 0, 1, or >1 outputs).

reading from Γ consumes (pops) a stack character; reading ε does not touch the stack.

writing to Γ pushes a character to the stack; writing ε does not touch the stack.

* Deterministic DFAs also exist; we won't see them.

* Often convenient to add a symbol # to the stack, so we know we reached the end.

e.g. $L = \{$correct open/close parentheses$\}$

Informally: push '(', pop ')', check stack never empty.

Diagram: