L19 2022-11-02

Recursion Theorem

th: Let t be computable. Then $\exists\, r$ computable by R st

$$r(x) = t(\langle R\rangle, x).$$

proof: let T be the TM computing T.

we build $M = A \circ B \circ T$ where

$A = P'_{\langle B \circ T\rangle}$ // P' prints $\langle B\circ T\rangle$ + original input.

$B =$ compute P_x
let $N = P_x \circ x$
write $\langle N\rangle$ + original input.

Incompleteness.

Let $\phi_{M,x} = \exists y\ \mathrm{CHECK}_A(M, x, y)$.
Obs $\phi_{M,x}$ true iff M accepts x.

Let S be the following TM:

1. obtain $\langle S\rangle$ via recursion thm.
2. compute $\psi = \neg \phi_{S,\varepsilon}$
3. accept if ψ has a proof.

Claim: ψ is true.

If ψ is false, then S accepts ε, then ψ has a proof, then ψ is true !!

Claim: ψ is not provable.

Because ψ is true, S does not accept ε, hence ψ does not have a proof.

This finishes the high-level proof, but we need to verify some details. We've seen step 1, but what about 2 & 3?