L16.

Arithmetic Hierarchy

def A language is of type Σ^i if it can be written as

$L = \{x : \exists y_1 \forall y_2 \cdots Q_i y_i \ \ \psi(x, y_1 \ldots y_i)\}$; ψ decidable.

Π^i if

$L = \{ \quad \forall$

Δ^i if $L \in \Sigma^i \cap \Pi^i$

$\forall \exists \equiv$ or $\equiv$ sum $\equiv \Sigma$; $\forall \equiv$ and $\equiv$ product $\equiv \Pi$.

e.g. $L =$ "TMs that don't stop" is a Π^1 language.

$L = \{\langle M \rangle : \forall x \forall y \ \ y$ stops $\Rightarrow \neg CHECK(M, x, y)\}$

e.g. $L =$ "TMs that always accept"

$L = \{\langle M \rangle : \forall x \exists y \ \ y$ accepts $\wedge \ CHECK(M, x, y)\}$

thm $A \leq_m B$ and B $\Sigma^i/\Pi^i/\Delta^i$ then A $\Sigma^i/\Pi^i/\Delta^i$.

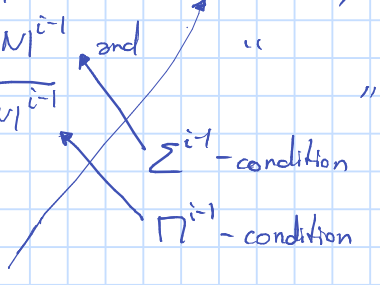
canonical language:

$UNI^i = \{\langle M, x \rangle$ st $M^{UNI^{i-1}}$ accepts $x\}$ is a Σ^i lang.

Indeed, $UNI^i = \{\langle M, x \rangle$ st $\exists y \ CHECK(M^{UNI^{i-1}}, x, y)\}$

$CHECK(M^{UNI^{i-1}}, x, y) \equiv$

"M accepts transcript y on input x and for each query q in y with answer yes $q \in UNI^{i-1}$ and no $q \in \overline{UNI^{i-1}}$"



does not count towards quantifier alternation: we are not quantifying a new string, simply iterating algorithmically over an existing one (y).