L15.

Arithmetic Hierarchy.

How can we classify languages stronger than recognizable?

We use an alternative def. of UNI.

Let $\text{CHECK} = \{ \langle M, x, y \rangle$ st M produces computation y on input $x \}$.

Then $\text{UNI} = \{ \langle M, x \rangle$ st M accepts $x \}$

$= \{ \langle M, x \rangle$ st $\exists y, \langle M, x, y \rangle \in \text{CHECK} \}$.

Obs $\overline{\text{UNI}} = \{ \langle M, x \rangle$ st $\neg(\exists y, \langle M, x, y \rangle \in \text{CHECK} \}$

$= \{ \langle M, x \rangle$ st $\forall y, \langle M, x, y \rangle \notin \text{CHECK} \}$.

What about $\text{EQ} = \{ \langle M, N \rangle$ that accept the same strings$\}$?

$\text{EQ} = \{ \langle M, N \rangle$ st $\forall x$

$(\exists y_M, y_N$ — $) \vee$

$(\forall y_M, y_N$ — $) \}$.

This language is of type $\forall \exists$.

prop. EQ is neither recognizable nor co-recognizable.

$f: \text{UNI} \to \text{EQ}$.

$\langle M, x \rangle \to \langle$ if input $= x$ run M, o/w reject,

if input $= x$ accept, o/w reject$\rangle$

$f: \overline{\text{UNI}} \to \text{EQ}$.

$\langle M, x \rangle \to \langle$ if input $= x$ run M, o/w reject,

reject$\rangle$

def A language is of type Σ^i if it can be written as

$L = \{ x : \exists y_1 \forall y_2 \cdots Q_i y_i \quad \varphi(x, y_1 - y_i) \}$; φ decidable.

Π^i if

$L = \{ \quad \forall$

Δ^i if $L \in \Sigma^i \cap \Pi^i$

* $\exists \equiv$ or $\equiv$ sum $\equiv \Sigma$; $\forall \equiv$ and $\equiv$ product $\equiv \Pi$.