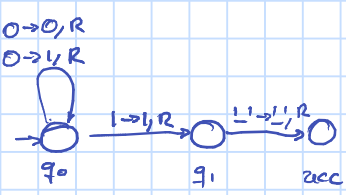
L13.

Notation (for the drill). Recall a TM configuration is a tuple (s, i, q) where s is the tape, i is the position of the head, and q is the state.

A more compact notation is $s_1 \dots s_{i-1} q s_i \dots s_m$.

e.g. if have TM



and input 001

then we reach the following configurations

t_0: $q_0 001$

t_1: $0 q_0 01$, $1 q_0 01$

t_2: $00 q_0 1$, $01 q_0 1$, $10 q_0 1$, $11 q_0 1$

t_3: $001 q_1$, $011 q_1$, $101 q_1$, $111 q_1$

t_4: 001_acc, 011_acc, 101_acc, 111_acc

Reductions

def A mapping reduction from A to B is a computable function f st $s \in A$ iff $f(s) \in B$.

def A is (mapping-) reducible to B ($A \leq_m B$) if there is a mapping reduction from A to B.

* why do we need "computable"?

thm. $A \leq_m B$ and B decidable $\Rightarrow$ A decidable.
equiv. A undecidable $\Rightarrow$ B undecidable.

proof. Let $f: A \to B$ and M_f a TM computing it.
Let M_B be a TM deciding B.

M_A. input: x.
run M_f on x.
run M_B

M_A accepts iff M_B accepts iff $M_f(x) \in B$ iff $f(x) \in B$ iff $x \in A$.
M_A always stops. ∎

e.g. $B = HALTB = \{\langle M_s \rangle : M \text{ halts on input } "s"\}$.
$A = HALT = \{\langle M, s \rangle : s\}$.

$f: A \to B$; $\langle M, s \rangle \mapsto \langle M_s \rangle$ where M_s is TM that writes s on tape and runs M.

This implies HALTB not decidable.

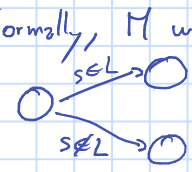
* obs thm also true when B recognizable $\Rightarrow$ A recognizable.

* obs we make a single call and we do not postprocess the input. Reductions that allow that are called Turing reductions.

Oracles.

A TM M with oracle L can query wheter $s \in L$ for any string s. Can do this many times.

Formally, M writes s on a special tape and has transitions



def A Turing-reducible to B ($A \leq_T B$) if $\exists M$ st M^B accepts A.

thm $A \leq_T B$ and B decidable, then A decidable.

proof: replace call to B with TM deciding B.

obs thm not true when B recognizable $\Rightarrow$ A recognizable.

⚠ Oracle TMs can be unexpectedly powerful.

e.g. if we give UNI as oracle, not only we can decide recognizable languages, we can also decide co-recognizable languages.