L10 2022-10-03

Models of TMs

th the following models recognize the same languages:

(1) PDAs with $K \geq 2$ stacks
(2) TMs with one-sided infinite tape (indexed by $\mathbb{N}$).
(3) TMs with two-sided infinite tape (indexed by $\mathbb{Z}$).
(4) TMs with K tapes.

proof sketch:

(3) → (4). (we'll see 2 tapes). position i encodes tape 1

position i and tape 2 position i. Also marks for which cells are being pointed to.

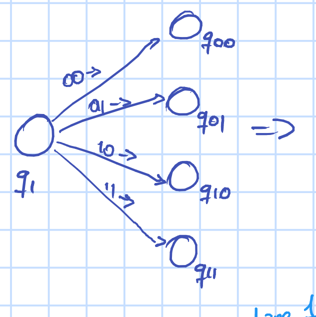
alphabet is $(\Gamma \times \{0,1\})^2$.

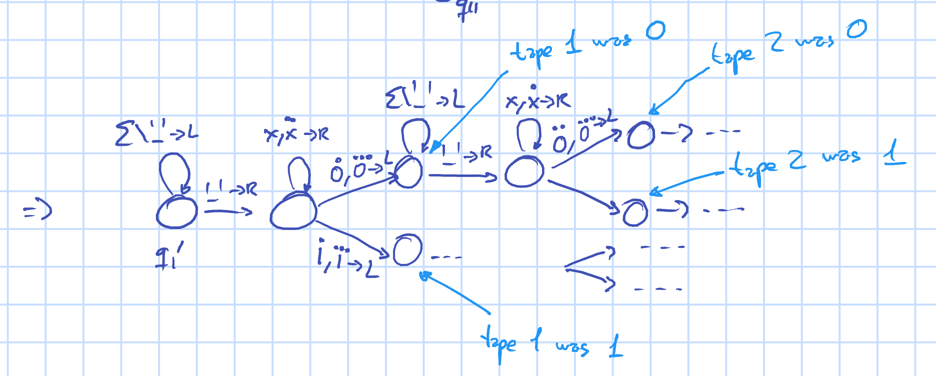
character in tape   is pointer here?

replace every state by a "cloud" of states, each encoding some information.

to simulate a move:
1. search for position of first pointer, record character (by moving to appropriate state in cloud)
2. " " second "
3. follow transition.
   (search first pointer, rewrite, move L/R, rewrite,
   search second " " " ")

More formally:





Nondeterministic TMs.

def: Usual, but δ is not necessarily a function and can have 0, 1, or more outputs.

∄ no empty transitions.

th: Det. and NonDet. TMs recognize the same languages.

proof sketch:

We simulate possible configurations of N in BFS order.

(obs if we did DFS we could fall into loop)

We use a TM with 3 tapes, where we store:
1: input
2: working tape
3: position in search tree.

For each position string in BFS order:
Copy input to working tape.
Simulate N, taking j-th choice in transition i if tape 3 has j in position i.

Other models TM equiv. to:

→ Cell automata
→ Lambda calculus
→ RAM machines
→ Uniform circuits
→ Quantum machines
→ Slime mold

Therefore TMs believed "universal". This conjecture is the Church-Turing thesis.

Undecidability.

Can TMs recognize/decide all languages? No!

th: there are more languages than TMs.

lemma: every TM can be represented with a binary string.

(just list its components; c.f. "serialization").

lemma: there is a bijection TMs ⟺ $\mathbb{N}$.

(list in increasing size order; break ties alphabetically).

lemma: there is no bijection Languages ⟺ $\mathbb{N}$.

(if you know set theory: Languages ⟺ $\mathcal{P}(\mathbb{N})$, which has cardinality ω_1, while $\mathbb{N}$ has cardinality ω_0).

this proves th. Let us see proof of lemma.

proof of lemma.

Proof by contradiction. Uses "diagonalization" technique.

Assume there is a bijection languages ⟺ $\mathbb{N}$.

Write table

strings

$$
\begin{array}{c|cccccc}
 & s_1 & s_2 & s_3 & \cdots & s_n & \cdots \\
\hline
L_1 & a_{11} & a_{12} & a_{13} & & & \\
L_2 & a_{21} & & & & & \\
L_3 & a_{23} & & & & & \\
\vdots & & & & & & \\
L_n & & & & & & \\
\vdots & & & & & &
\end{array}
$$

languages

$a_{ij} = 1$ if set i contains element j
$\quad\;\; 0$ o/w.

Consider language $L = \{ s_i \in L \text{ iff } s_{ii} \notin L_i \}$

Its row is $(1 - a_{ii})_i$, that is the opposite of the diagonal.

But such a row does not appear in our table!!
(if it were L_m, then $a_{mm} = 1 - a_{mm}$!!).

⌗