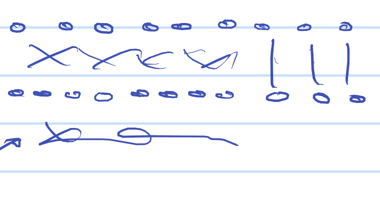
Branchwidth. th: $\exists$ algorithm st if F has $bw(V\|H) \leq k$ then algo run in time $2^{O(k)} \cdot poly(n)$.

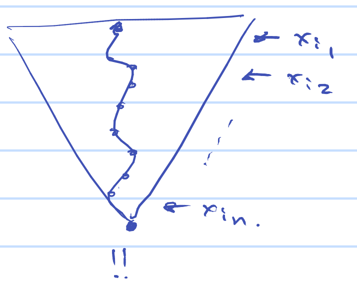
based on DP, i.e. $elim(F, (x_{i_1}, x_{i_2} \dots x_{i_n}))$ $i_1 \dots i_n$ order given by branch decomposition.

resolution steps with pivot x_{i_1}

x_{i_2}



(low: $\dfrac{A \vee x_{i_1} \quad B \vee \bar{x}_{i_1}}{A \vee B}$).



every path from clause to !!
sees a subset of $\{x_{i_1} \dots x_{i_n}\}$ as
res. variables in this order

(i.e. order same for every path).

def Resolution proof with this property (order same in every path) is called "ordered resolution".

th: $\exists$ F family of formulas st F has short (poly) resolution proofs (even tree-like).

but requires ordered resolution proofs of length $\exp(|F|)$.

produced by DPLL

open: is there smaller measure $\mu(F) \leq bw(V\|H)$. that leads to algorithm that requires full power of resolution?

$\hookrightarrow$ like CDCL, because CDCL poly-simulate resolution.

(cliquewidth?) Book "Parameterized Algorithms" $\to$ discussion of parameters.

Backdoors. Informally: backdoor set $B \subseteq Vars(F)$ st after assigning B, resulting formula becomes easy.

Backdoors defined with a class of formulas $\mathcal{C}$ in mind.

e.g. $\mathcal{C}$ = 2SAT, Horn SAT, ...

$\leftarrow$ ≤ 1 positive literal.

Properties that we want from $\mathcal{C}$:

- $\to$ $\mathcal{C}$ efficiently solvable ($\exists$ poly-time algo A st. $F \in \mathcal{C} \to A$ decide F in poly-time).
- $\to$ $\mathcal{C}$ efficiently recognizable (" A decide if $F \in \mathcal{C}$ in poly-time).
  - $\hookrightarrow$ implied by eff. solvable. run $A(F)$ for poly-many steps. if not finished $F \notin \mathcal{C}$.
- $\to$ $\mathcal{C}$ self-reducible ($F \in \mathcal{C} \to F\restriction_{x=b} \in \mathcal{C}$ $\forall x \in Var(F)$).
- $\to$ $\mathcal{C}$ downwards-closed ($F \in \mathcal{C} \to F \setminus C \in \mathcal{C}$ $\forall C \in F$).

claim: 2SAT, Horn SAT satisfy properties:

$\hookrightarrow$ at most one positive literal in every clause.

2SAT: ✓ ✓ ✓ ✓.

HornSAT: ✓ ✓ ✓ ✓.

$\bar{x} \vee \bar{y}$, $\bar{x} \vee z$, w. Horn.

$x \vee y$, $\bar{x} \vee z$, $\bar{w}$ not Horn.

$\bar{x}_1 \vee \bar{x}_2 \vee \dots \bar{x}_k \vee y$

$(x_1 \wedge x_2 \wedge \dots \wedge x_k) \to y$.

def. $B \subseteq Vars(F)$ weak backdoor if $\exists \alpha \in \{0,1\}^B$ st $F\restriction_{B=\alpha} \in \mathcal{C}$.

def. $B \subseteq Vars(F)$ strong backdoor if $\forall \alpha \in \{0,1\}^B$: $F\restriction_{B=\alpha} \in \mathcal{C}$.

obs B strong back $\Rightarrow$ B weak backdoor.

obs $B = Vars(F)$ always strong backdoor. (assuming always false $\in \mathcal{C}$, always true $\in \mathcal{C}$).

obs B weak backdoor, F sat then can find β satisfying assignment for F in time $2^{|B|} \cdot poly(|F|)$.

Known

algorithm: for $\alpha \in \{0,1\}^B$:

find β' sat. assignm. for $F\restriction_{B=\alpha}$.

if $\beta' \exists$: $\alpha \cup \beta'$ sat. ass. for F.

o/w: next α.

obs B strong backdoor, can decide F in time $2^{|B|} \cdot poly(F)$.

for $\alpha \in \{0,1\}^B$:

decide if $F\restriction_{B=\alpha}$ $\in \mathcal{C}$ sat/unsat.

if sat: return sat.

return unsat.

but... how to find backdoors?

prop. F has backdoor of size $b \leq n/2$ $\to$ decide F in time $\left(\frac{n}{b}\right)^{O(b)}$.

proof: try all possible backdoors in increasing size order.

for $b' = 1 \dots n/2$:

for $B \subseteq \binom{[n]}{b'}$

subsets of $[1 \dots n]$ of size b.

try to decide F, assuming B backdoor.

(ie: run previous algorithm; if time out: not backdoor).

if finishes correctly: ✓.

$\binom{n}{b} \leq \left(\frac{n}{b}\right)^b$

running time: $\displaystyle\sum_{b'=1}^{\cancel{n/2}\ b} \binom{n}{b'} \cdot 2^{b'} \cdot poly(n) = O\left(\binom{n}{b}\right) = \left(\frac{n}{b}\right)^{O(b)}$.

backdoors st don't need to try all possible $2^{|B|}$ assignments?

def. deleting variables from F: $F \setminus B = \{C \setminus B : C \in F\}$.

$F = \{x_1 \vee x_2, \bar{x}_2 \vee x_3, x_1 \vee x_3\}$.

def. $B \subseteq Vars(F)$ deletion backdoor if $F \setminus B \in \mathcal{C}$.

$F \setminus \{x_2\} = \{x_1, x_3, x_1 \vee x_3\}$.

prop. $\mathcal{C}$ downwards-closed ($F \setminus C$ still in $\mathcal{C}$). B del. backdoor $\Rightarrow$ B strong backdoor.

proof. B del. back. $\forall \alpha \in \{0,1\}^b$ want to show $F\restriction_{B=\alpha} \in \mathcal{C}$.

$C \in F$ $\begin{cases} C \text{ sat by } \alpha \to C\restriction_{B=\alpha} \notin F\restriction_{B=\alpha}. \\ \text{every lit. in } C \cap B \text{ is falsified by } \alpha \to C \setminus B \in F\restriction_{B=\alpha}. \end{cases}$

$F\restriction_{B=\alpha} \subseteq F \setminus B \in \mathcal{C}$, by downwards-closed $F\restriction_{B=\alpha} \in \mathcal{C}$.

prop. if B strong 2-SAT back $\to$ B del. 2-SAT back.

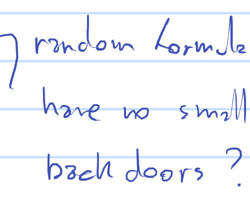
HornSAT HornSAT.

in fact this is true for any $\mathcal{C}$ st $\exists$ property P st $F \in \mathcal{C}$ iff $\forall C \in F$ $P(C)$ true.

$\forall \alpha$ $F\restriction_\alpha \in$ 2-SAT. $F \setminus B \in$ 2-SAT.

proof: show $P(C \setminus B)$ true for all $C \setminus B \in F \setminus B$.

fix. $C \setminus B$. let α assignment that fals. $C \cap B$.



consider $F\restriction_{B=\alpha}$. $C\restriction_\alpha = C \setminus B$, $F\restriction_{B=\alpha} \in \mathcal{C} \Rightarrow P(C \setminus B)$.

e.g. [$\in$ 2SAT $\Rightarrow$ $C \setminus B$ has 2 literals.]

$P(C \setminus B)$ true $\forall$ $\Rightarrow$ $F \setminus B \in \mathcal{C}$.

[pick $8n$ clauses of width 3 unif. random. $\to$ unsat whp. hard whp.

] random formulas have no small backdoors?

vars of size $n/2$ $\to$ F over $n/2$ variables, close to random formula $\to$ unsat whp hard whp.]