algo for [AR11] builds on DP algorithm.

def. given formula F and variable x, $\text{elim}(F,x)$ is the result of resolving away any instances of x.

formally: $\text{elim}(F,x) = \{\text{Res}(C,D) : C,D \in F, x \in C, \bar{x} \in D\}$
$\cup \{E \in F : x \notin \text{vars}(E)\}$.

properties.

(i) commutativity: $\text{elim}(F,x,y) = \text{elim}(F,y,x)$.

proof: for each $C \in \text{elim}(F,x,y)$ check cases depending on whether x,y contained in ancestors of C.

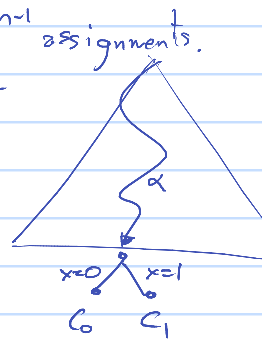
obs this means $\text{elim}(F,S)$, S set, is well-defined.

(ii) equisatisfiability: F sat iff $\text{elim}(F,x)$ sat.

$\Rightarrow$) ok by soundness of resolution.

$\Leftarrow$). Assume F unsat.

Consider decision tree with all 2^{n-1} assignments. We'll show that each α assignment to $\text{Vars}(F) \setminus x$ falsifies some clause in $\text{elim}(F,x)$.



Indeed, because F unsat, for each
$\alpha_0 = \alpha \cup \{\bar{x}\}$, $\alpha_1 = \alpha \cup \{x\}$
$\exists\, C_0, C_1 \in F$ st $\alpha_i(C_i) = 0$.
if $x \notin C_0$ or $x \notin C_1$, then they carry on to $\text{elim}(F,x)$, and falsified by α,
o/w $C = \text{Res}(C_0, C_1) \in \text{elim}(F,x)$ and falsified by α.

algorithm [Davis, Putnam].

compute $\text{elim}(F, \text{vars}(F))$. answer $\begin{cases} \text{sat if } \top \\ \text{unsat if } \bot \end{cases}$.

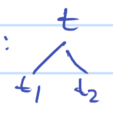
obs runtime of DP depends on order: even if end result is the same, intermediate steps might have wildly different sizes.

idea behind AR11 is to use branch decomposition to find a good order for DP.

proof (of AR11).

Fix branch decomposition T. Traverse T upwards (leaves $\Rightarrow$ root). We will assign to each vertex $t \in T$ a set of clauses $C(t)$, st
$\text{vars}(C(t)) \subseteq \text{Cut}(t)$.

leaves: $C(t) = \begin{cases} \emptyset & \text{if } E(t) \text{ has a variable not appearing elsewhere.} \\ E(t) & \text{o/w} \end{cases}$

update: 

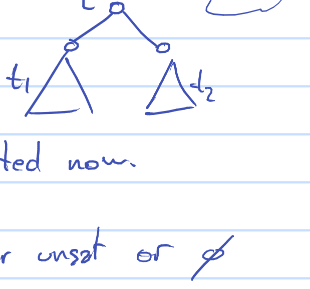
just resolve away any extra variables. (i.e. in $\text{Cut}(t_1)$ or $\text{Cut}(t_2)$ but not t).

formally: $C(t) = \text{Elim}\big(C(t_1) \cup C(t_2),\ \text{cut}(t_1) \cup \text{cut}(t_2) \setminus \text{cut}(t)\big)$

claim: $\text{vars}\big(C(t)\big) \subseteq \text{Cut}(t)$. ok by construction.

claim: $F \setminus E(t) \cup C(t) = \text{Elim}(F, \underbrace{V(t) \setminus \text{Cut}(t)}_{\text{vars only in subtree}})$.

leaves OK. induction:
-> $V(t) \setminus \text{Cut}(t)$ disjoint in different subtrees.
-> $x \in \text{cut}(t_1) \cup \text{cut}(t_2) \setminus \text{cut}(t)$
then x only in subtree.
-> x only in subtree. if only in sub_1 or sub_2, already eliminated. o/w eliminated now.



at root we either have $\bot$ and answer unsat or $\emptyset$ and answer sat.

obs because there are 3^k clauses over K vars, we have $|C(t)| \le 3^{|\text{Cut}(t)|} \le 3^k$.

hence running time is $\le |F| \cdot (3^k)^2 \le \text{poly}(n) \cdot \exp(bw)$.