We saw CDCL simulates resolution if we have nondet. decisions, frequent restarts, asserting learning, no deletions. What about other parameters?

-> Decisions:
- -> random: can p-sim bounded-width res (saw).
- -> deterministic: cannot p-sim res unless P=NP
- -> static (fix order at the beginning of the algorithm & never change): cannot p-sim res.
- -> current heuristics (VSIDS-like): cannot p-sim res.

-> Restarts:
- -> open if no restarts at all can p-sim res.
- -> open if current heuristics (LBD) can p-sim res.

-> Learning:
- -> overhead is lower with "decision" than "1UIP".
- -> open if poly separation between decision and 1UIP.
- -> open if poly separation between res and CDCL.

-> Deletions:
- -> in resolution, it is enough to remember n clauses even if $L = \exp(n)$. open if also true for CDCL.
- -> if remember $\ll n$ clauses cannot p-sim res. ⟶ we'll see this another day.
- -> if only remember narrow (small width) clauses cannot p-sim res.
- -> open if remember like current heuristics (small pseudo-width / LBD) can p-sim res. (conjecture: no).

Can we get results that only depend on the structure of the formula?

def variable incidence hypergraph of a formula F (VIH) is $H(V,E)$; $V = Vars(F)$;

$$E = \{Vars(C) : C \in F\}.$$

def variable incidence graph of F (VIG) is primal graph of VIH. (hyperedge -> clique).
iow: $E = \{(x,y) : \exists C \in F \; \{x,y\} \subseteq Vars(C)\}$.

def clause-variable incidence graph of F (CVIG) is bipartite graph $V = Vars(F) \cup F$;

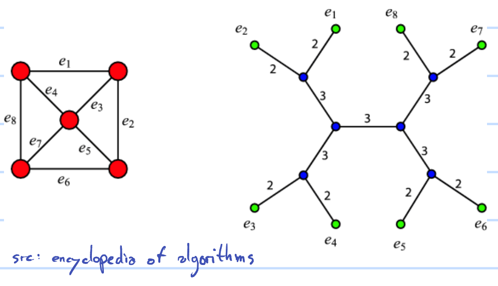
$$E = \{(x,C) : x \in Vars(C)\}.$$

obs $(CVIG)^2$ contains VIG.

obs these definitions do not depend on polarity! a hard formula and the same formula with no negations (trivially sat) have the same graph. we'll only be able to prove upper bounds.

Often treewidth useful graph measure. We'll use branchwidth, which is at most a factor 2 apart. Informally a branch decomposition is a hierarchical clustering of edges.

def branch decomposition of $G=(V,E)$
T rooted binary tree with leaves identified with E.
given vertex $x \in T$, $E(x)$ = edges in subtree of x.
$Cut(x) = V(E(x)) \cap V(\overline{E(x)})$.



src: encyclopedia of algorithms

def branch width.

$$bw(T) = \max_x |Cut(x)|$$

$$bw(G) = \min_T bw(T).$$

th [Robertson, Seymour] Exists deterministic algo st if $bw(G) = k$ outputs branch decomposition of G of width $O(k)$ and runs in time $poly(|G|) \cdot \exp(k)$.

th [Gottlob, Scarcello, Sidori '02] exists deterministic algo for SAT in time $poly(F) \cdot \exp(tw(VIG))$.

th [Alekhnovich, Razborov '11] exists deterministic algo for SAT in time $poly(F) \cdot \exp(bw(VIH))$.

obs SAT is not only a property of the graphs but of the signs too; hence Courcelle's thm does not apply.

obs second thm can be significantly better:

$F = \{x_1 \vee \dots \vee x_n\}$ has $tw(VIG) = n$
but $bw(VIH) = 1$.