def. Algorithm A poly-simulates resolution if $\forall F$ formula, $\pi = (C_1 \dots C_L)$ proof of F we have $A(F)$ runs in time $\leq poly(|F|, L = |\pi|)$.

$\to$ we have seen this, but A is nondeterministic.
if A deterministic and A p-sim res. then $P = NP$.

th: [BKS 04] non-greedy CDCL, nondeterministic decisions, adversarial learning, always restarts, never delete $\to$ p-simulates resolution.

allowed to not do unit propagations even if it should.
allowed to continue past conflict.

proof: let $\pi = (C_1 \dots C_L)$ be a proof. plan: learn $C_1 \dots C_L$ in order.

let $C_i = \ell_1 \vee \ell_2 \vee \dots \vee \ell_n$.

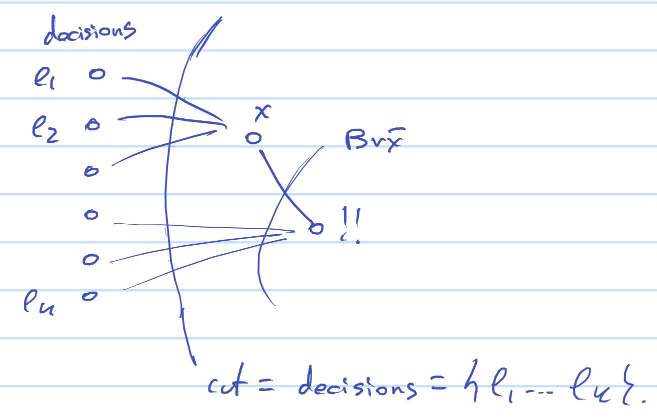
assign trail: $\alpha = \overline{C_i} = \overline{\ell_1}^d \wedge \overline{\ell_2}^d \wedge \dots \overline{\ell_n}^d = (\ell_1 \stackrel{d}{=} 0, \ell_2 \stackrel{d}{=} 0, \dots \ell_n \stackrel{d}{=} 0)$ do no UPs / conflict detection.

immediately get conflict. why?

$C_i = Res(\underbrace{A \vee x}_{\in D}, \underbrace{B \vee \bar{x}}_{\in D})$
$\overset{\shortparallel}{A \vee B}$

$A \subseteq C_i$ $\quad$ $\cancel{A} \vee x \to$ unit clause
$B \subseteq C_i$ $\quad$ $\cancel{B} \vee \bar{x} \to$ unit clause.

unit propagate x
conflict with $\bar{x}$.



cut = decisions = $\{\ell_1 \dots \ell_n\}$.

only one possible learned clause: $\overline{\alpha} = C_i$. as we wanted, $D = D \cup \{C_i\}$.

after learning C_i: restart ($\alpha = \emptyset$). move to next clause.

to learn C_i we do $\leq n$ steps: time is $\leq n \cdot L = poly(|F|, |\pi|)$.

th [PD 11]: CDCL (usually greedy), with nondet. decisions, asserting learning, always restart, never delete $\to$ p-simulates resolution.

after learning C, backtracking, then $\alpha(C)$ unit.
e.g. decision 1UIP.

idea: no need to learn all C_i.
if have D st $\forall \alpha$ (trail) st $\alpha(C_i)$ unit propagates ℓ
we also have that D unit propagates ℓ.] $\exists C_j \in D$ $\alpha(C_j)$ unit propagates ℓ.
depend on α / ℓ.
then act as if we had learned C_i.

def: D absorbs C if for every literal $\ell \in C$ it holds that setting trail $\alpha = \overline{C \setminus \ell}$ then $D \vdash_1 \ell$. (or $D \vdash_1 !!$).
$\hookrightarrow$ run unit propagation iteratively

example: $D = a \vee \bar{b}, \quad b \vee c, \quad \bar{a} \vee \bar{b} \vee d \vee e$.

claim: D absorbs $a \vee c$. ✓ $= Res(a \vee \bar{b}, b \vee c)$.

a. $\quad \alpha = \overline{(a \vee c \setminus a)} = \bar{c} = \{c = 0\}$.
$D|\alpha \vdash_1 (b = 1, a = 1)$

$a \vee \cancel{\bar{b}}$
$b \vee \cancel{c}$
$\bar{a} \vee \bar{b} \vee d \vee e$

c. $\quad \alpha = \bar{a} = \{a = 0\}$.
$\bar{b} = 1 \iff b = 0$
$c = 1$

$\cancel{a} \vee \bar{b}$
$\cancel{b} \vee c$
$\cancel{\bar{a} \vee \bar{b} \vee d \vee e}$

claim: D does not absorb ✗ $c \vee d \vee e$. $= Res(b \vee c, \bar{a} \vee \bar{b} \vee d \vee e)$.

c ?? $\quad \alpha = \bar{d} \wedge \bar{e} \to \times$.
d
e

$F|_\alpha \vDash c$ implies
but $F|_\alpha \not\vdash_1 c$ not unit implies
(does not follow from unit propagation).

$a \vee \bar{b}$
$b \vee c$
$\bar{a} \vee \bar{b} \vee \cancel{d} \vee \cancel{e}$

2-CNF:
no more UP.

[detour: Knowledge Representation

given F CNF, want to build a data structure st can quickly check if $F \vDash C$.

naive solution: store all possible Cs.

smarter way: only store Cs not absorbed by data structure.

to find $F \vDash C$: set $\alpha = \bar{C}$. if $C \in D \to$!!

if $C \notin D$ but absorbed
then $\alpha \setminus \alpha_k$ $<$ propagate $\overline{\alpha_k}$ / had set α_k) conflict.
any lit. in α.
]

properties of absorption:

(i) sound: if D absorbs C then $D \vDash C$.
(ii) closed: if $C \in D$ then D absorbs C.
(iii) clause-monotone: if D absorbs C, $C \subseteq C'$ then D absorbs C'.
(iv) DB-monotone: if D absorbs C, $D \subseteq D'$ then D' absorbs C.

proof. (i). $D \vDash C$ iff $D \wedge \bar{C} \vDash$!!

assume D absorbs C. then $\forall a \in C$: $\alpha = \overline{C \setminus a}$ $\quad D \wedge \alpha \vdash_1 a. \Rightarrow$

$D \wedge \alpha \vDash a$.

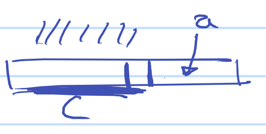
$D \wedge \bar{C} = \underbrace{D \wedge \alpha}_{\vDash a} \wedge \bar{a}$

$D \wedge \alpha \wedge \bar{a} \vDash a$.
$D \wedge \alpha \wedge \bar{a} \vDash \bar{a}$) $D \wedge \alpha \wedge \bar{a} \vDash$!!

(ii) $C \in D \to D$ absorbs C.

let $a \in C$. set $\alpha = \overline{C \setminus a}$, do UP. because $C \in D$:
C is unit on trail $\alpha \to$ propagate a. $\to D \vdash_1 a$.

(iii) D abs C, $C \subseteq C'$ then D abs C'.



let $a \in C'$. set $\alpha = \overline{C' \setminus a}$.

$\to$ if $a \notin C$ consider $\beta = \alpha \cap \bar{C}$, $\gamma = \beta \setminus \beta_k$.

C absorbed: $D \wedge \gamma \vdash_1 \overline{\beta_k}$.

$D \wedge \alpha \vdash_1 \overline{\beta_n}$ $\quad$ $\beta_n \in \beta \subseteq \alpha$. $\quad$ $D \wedge \alpha \vdash_1 \beta_k$.
$\alpha \supseteq \gamma$

$D \wedge \alpha \vdash_1$!!

$\to$ if $a \in C$ consider $\beta = \alpha \cap \bar{C}$

C absorbed: $D \wedge \beta \vdash_1 a$
$D \wedge \alpha \vdash_1 a$.

(iv): D abs C, $D \subseteq D'$ then D' abs C. //.

proof plan (of CDCL p-sim res): $\pi = (C_1 \dots C_L)$.

invariant: at time $i' = i \cdot n^4$ $D_{i'}$ absorbes all clauses C_j: $j < i$.

never stop absorbing: enough to prove at time i' $D_{i'}$ absorbes C_i. (by prop (iv)).

if $C_i \in F$ then C_i already absorbed (by prop (ii)).

o/w $C_i = Res(A \vee x, B \vee \bar{x})$ $\quad$ $A \vee x$ absorbed by D
$B \vee \bar{x}$ absorbed by D.

would like to set $\alpha = \overline{C_i}$, learn C_i.
but we have UPs now, maybe we cannot do this!

$x \vee y \in D$.

$\alpha = \{x = 0 \wedge y = 0\}$.

$\{x \stackrel{d}{=} 0, y = 1\}$.