CDCL: learning.

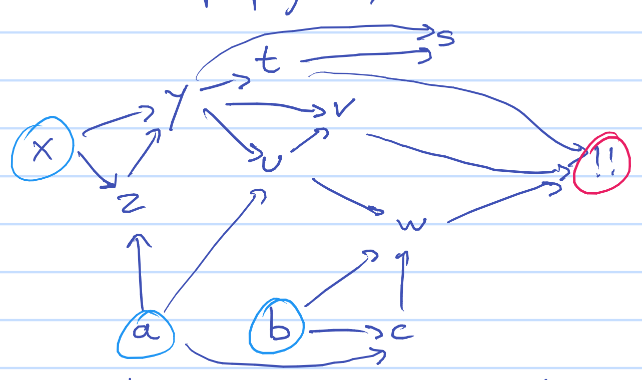
decision learning scheme: learn opposite of decisions in trail.

def: Conflict graph: vertices are variables

edge $x \to y$ if clause containing x was used to unit propagate y.

Example.

C_1 $\bar{x} \vee \bar{a} \vee z$
C_2 $\bar{x} \vee \bar{z} \vee y$
C_3 $\bar{y} \vee t$
C_4 $\bar{y} \vee \bar{a} \vee u$
C_5 $\bar{y} \vee \bar{u} \vee v$
C_6 $\bar{y} \vee \bar{t} \vee s$
C_7 $\boxed{\bar{u} \vee \bar{b} \vee \bar{c}} \vee w$ E'
C_8 $\boxed{\bar{t} \vee \bar{v}} \vee \bar{w}$!! E
C_9 $\bar{a} \vee \bar{b} \vee c$



α:	$a \overset{d}{=} 1$	$b \overset{d}{=} 1$	$c \overset{C_9}{=} 1$	$x \overset{d}{=} 1$	$z \overset{C_1}{=} 1$	$y \overset{C_2}{=} 1$	$t \overset{C_3}{=} 1$	$u \overset{C_4}{=} 1$	$v \overset{C_5}{=} 1$	$s \overset{C_6}{=} 1$	$w \overset{C_7}{=} 1$	$\overset{C_8}{!!}$
	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	α_{11}	

C_8 conflict. with literal α_{11}. $C_8(\alpha) = 0$ $C_8(\alpha_{\le 10}) = \bar{w}$.

C_7 unit propagates w. $\alpha_{\le 10}$ $C_7(\alpha_{\le 10}) = w$

$C_8 = E \vee \bar{w}$.
$C_7 = E' \vee w$.

Resolve C_7, C_8 $\dfrac{E \vee w \quad E' \vee w}{E \vee E'}$.

E, E' no opposite literals. $E(\alpha_{\le 10}) = 0$
$E'(\alpha_{\le 10}) = 0$.

$\exists x \in Vars(E)$ want to show: either $x \in E, x \in E'$ or $\bar{x} \in E, \bar{x} \in E'$.
$\exists x \in Vars(E')$

if $x \in Vars(E)$ and $\beta(E) = 0$

then β assigns some value to x.

value is opposite to how x appears in E.

if $x \in E \to \beta(x) = 0$
if $\bar{x} \in E \to \beta(x) = 1$.

$\forall$ literal ℓ in a clause $\underset{E}{:}$ if an assignment β fals. E : $\beta(E) = 0$.

then $\ell \in \beta$, $\beta(\ell) = 0$.

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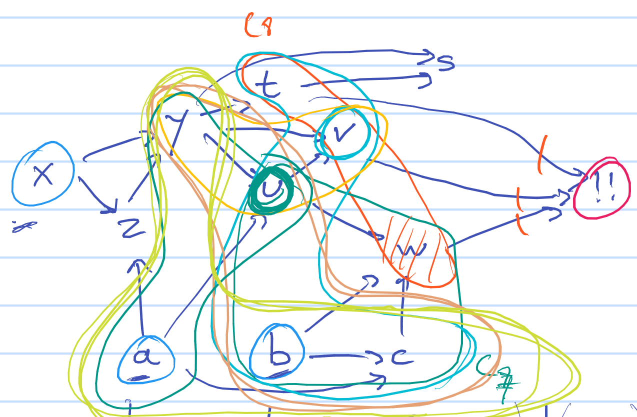

α:	$a \overset{d}{=} 1$	$b \overset{d}{=} 1$	$c \overset{C_9}{=} 1$	$\cancel{x \overset{d}{=} 1}$	$\cancel{z \overset{C_1}{=} 1}$	$\cancel{y \overset{C_2}{=} 1}$	$\cancel{t \overset{C_3}{=} 1}$	$\cancel{u \overset{C_4}{=} 1}$	$\cancel{v \overset{C_5}{=} 1}$	$\cancel{s \overset{C_6}{=} 1}$	$\cancel{w \overset{C_7}{=} 1}$	$\overset{C_8}{!!}$
	α_1	α_2	α_3	$\cancel{\alpha_4}$	$\cancel{\alpha_5}$	$\cancel{\alpha_6}$	$\cancel{\alpha_7}$	$\cancel{\alpha_8}$	$\cancel{\alpha_9}$	$\cancel{\alpha_{10}}$	$\cancel{\alpha_{11}}$	

$C_7, C_8 \to \underbrace{\bar{u} \vee \bar{b} \vee \bar{c} \vee \bar{t} \vee \bar{v}}_{D_7}$ $D_7, C_5 \to \underbrace{\bar{u}, \bar{b}, \bar{c}, \bar{t}, \bar{y}, \cancel{\bar{v}}}_{D_6}$ $D_6, C_4 \to \underbrace{\bar{b} \vee \bar{c} \vee \bar{t} \vee \bar{y} \vee \bar{a}}_{D_5}$

$D_5, C_3 \to \underbrace{\bar{b} \vee \bar{c} \vee \bar{y} \vee \bar{a}}_{D_4}$ D_4, C_2 $\underbrace{\bar{b} \vee \bar{c} \vee \bar{a} \vee \bar{x} \vee \bar{z}}_{D_3}$ D_3, C_1 $\underbrace{\bar{b} \vee \bar{c} \vee \bar{a} \vee \bar{x}}_{D_2}$

$D_2, C_9 \to \underbrace{\bar{b} \vee \bar{a} \vee \bar{x}}_{D_1}$ D_1 = negation of decisions.

$F|_{x=1, a=1, b=1} \to$!! $F \wedge x \wedge a \wedge b = \perp$

$A \wedge B = \perp$
$A \to \bar{B}$.

$F \to \neg(x \wedge a \wedge b)$

$F \to \bar{x} \vee \bar{a} \vee \bar{b}$.

conflict analysis: analyse conflict graph, find which clause to learn.

C_1 $\bar{x} \vee \bar{a} \vee z$
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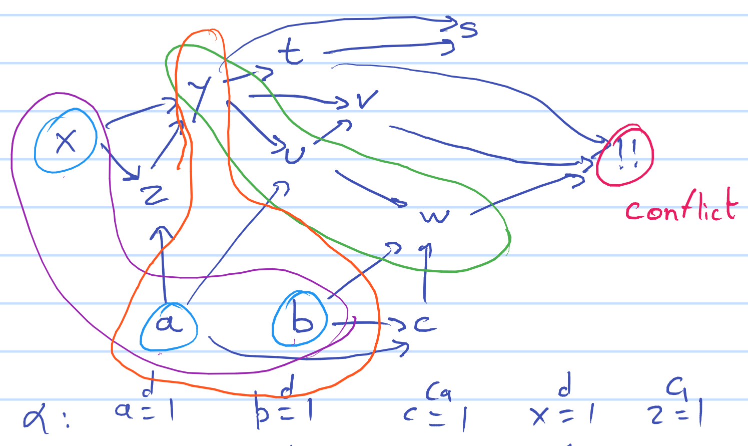

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	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	α_{11}	
dl:	1	2	2	3	3	3	3	3	3	3	3	3

we can always learn negation of decisions.

prop: if X is a cut separating decisions and conflict then possible to learn

$X = \{x_1, \dots, x_k\}$.

$\alpha(x_1) = b_1 \dots \alpha(x_k) = b_k$.

claim: set $\beta(x_1) = b_1 \dots \beta(x_k) = b_k$.
+ unit propagation $\to$ conflict.

claim: learn $\bigvee_{i=1}^{k} (x_i = \bar{b}_i)$

$F \wedge (x_1 = b_1 \dots \wedge x_k = b_k) = \perp$

$F \to \neg($ —— $)$.

practice: UIP cuts "unique implication point".

cut st only one variable of maximum decision level.

in example: (a, b, y) UIP.

prop: $\{$decisions$\}$ always a UIP. (all decs in different levels).

def First UIP learning: start with conflict. do resolution until reach UIP.
then learn that clause.